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ON THE EXTRACTION OF NON-PERTURBATIVE EFFECTS IN THE FRAGMENTATION FUNCTIONS OF HEAVY QUARKS IN e^+e^- ANNIHILATION [a]

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In this talk, we consider the computation of D and B fragmentation functions in e^+e^- annihilation. We present an improved differential cross section that merges together the $\mathcal{O}(\alpha_s^2)$ fixed-order calculation and the next-to-leading-logarithmic resummed one. We compare the results of fitting present data using the next-to-leading-log cross section, the $\mathcal{O}(\alpha_s^2)$ fixed-order and the improved one.

1   Introduction

The interest in refining our knowledge of the fragmentation functions for heavy quarks (HQFF) in e^+e^- annihilation stems mainly from the possibility of using them to improve our understanding of heavy-flavour hadroproduction and photoproduction. In fact, this is a quite clean environment where the impact of non-perturbative effects can be studied.

The aim of this talk is to present new results on the size of non-perturbative effects on heavy-flavour production in e^+e^- annihilation.

The outline of the talk is as follows. We first give a brief introduction of the fragmentation-function formalism and we introduce an improved differential cross section, which merges together the $\mathcal{O}\left(\alpha_s^2\right)$ fixed-order result with the next-to-leading-logarithmic resummed one. We parametrize the non-perturbative contributions to the differential cross section using the Peterson form [1]. We present some fits to recent data from which we extract the non-perturbative part. Then we conclude with some comments on the obtained results.

2   Theoretical framework

We consider the inclusive production of a heavy quark Q of mass m, in the process

$$e^+e^- \to Z/\gamma\,(q) \to Q\,(p) + X \,, \tag{1}$$

where q and p are the four-momenta of the intermediate boson and of the final quark. We define x_E the scaled energy of the final heavy quark, and x_p the normalized momentum fraction

$$x_E = \frac{2\,p\cdot q}{q^2}\,, \qquad x_p = \frac{\sqrt{x_E^2 - \rho}}{\sqrt{1-\rho}}\,, \qquad \text{where}:\quad \rho = \frac{4\,m^2}{E^2} \quad\text{and}\quad E = \sqrt{q^2}\,. \tag{2}$$

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The inclusive cross section for the production of a heavy quark can be written as a perturbative expansion in α_s

$$\frac{d\sigma}{dx_p} = \sum_{n=0}^{\infty} a^{(n)}(x_p, E, m, \mu) \, \bar{\alpha}_s^n(\mu) \,, \qquad \bar{\alpha}_s(\mu) = \frac{\alpha_s(\mu)}{2\pi} \,, \tag{3}$$

where μ is the renormalization scale.

If $\mu \approx E \approx m$, the truncation of Eq. (3) at some fixed order in the coupling constant can be used to approximate the cross section. An $\mathcal{O}\left(\alpha_s^2\right)$ fixed-order calculation for the process (1) is available [2, 3, 4], so that we can compute the coefficients of Eq. (3) at the $\mathcal{O}\left(\alpha_s^2\right)$ level. We define the fixed-order (FO) result as

$$\left.\frac{d\sigma}{dx_p}\right|_{\text{FO}} = a^{(0)}(x_p, E, m) + a^{(1)}(x_p, E, m) \, \bar{\alpha}_s + a^{(2)}(x_p, E, m) \, \bar{\alpha}_s^2 \,, \tag{4}$$

where we have taken $\mu = E$ for ease of notation, and, from now on, $\bar{\alpha}_s \equiv \bar{\alpha}_s(\mu)|_{\mu=E}$.

If $E \gg m$, large logarithms of the form $\log\left(E^2/m^2\right)$ appear in the differential cross section (3) to all orders in α_s. In this limit, if we disregard all power-suppressed terms of the form m^2/E^2, the inclusive cross section can be organized in the expansion

$$\left.\frac{d\sigma}{dx}\right|_{\text{res}} = \sum_{n=0}^{\infty} \beta^{(n)}(x) \, (\bar{\alpha}_s L)^n + \bar{\alpha}_s \sum_{n=0}^{\infty} \gamma^{(n)}(x) \, (\bar{\alpha}_s L)^n + \bar{\alpha}_s^2 \sum_{n=0}^{\infty} \delta^{(n)}(x) \, (\bar{\alpha}_s L)^n + \dots \,, \tag{5}$$

where $L = \log\left(E^2/m^2\right)$ and x stands for either x_E or x_p, since the two variables differ by power-suppressed effects.

The leading-logarithmic (LL) and next-to-leading-logarithmic (NLL) cross sections are given, respectively, by

$$\left.\frac{d\sigma}{dx}\right|_{\text{LL}} = \sum_{n=0}^{\infty} \beta^{(n)}(x) \, (\bar{\alpha}_s L)^n \,, \qquad \left.\frac{d\sigma}{dx}\right|_{\text{NLL}} = \sum_{n=0}^{\infty} \beta^{(n)}(x) \, (\bar{\alpha}_s L)^n + \bar{\alpha}_s \sum_{n=0}^{\infty} \gamma^{(n)}(x) \, (\bar{\alpha}_s L)^n \,. \tag{6}$$

The $\delta^{(n)}$ coefficients of Eq. (5) define the NNLL terms, that are, as of now, not known. The technique to resum these large logarithms is well established and more details can be found in Ref. [5].

The expansion of the NLL differential cross section up to order α_s^2, that we call the truncated NLL (TNLL from now on), is given by

$$\left.\frac{d\sigma}{dx}\right|_{\text{TNLL}} = \beta^{(0)}(x) + \bar{\alpha}_s \left(\gamma^{(0)}(x) + \beta^{(1)}(x) L\right) + \bar{\alpha}_s^2 \left(\gamma^{(1)}(x) L + \beta^{(2)}(x) L^2\right) \,, \tag{7}$$

and does not coincide with the massless limit of the FO calculation: a term of order α_s^2, not accompanied by logarithmic factors, may in fact survive in the massless limit of the FO result. In the HQFF approach, this is a NNLL effect, and therefore it is not included at the NLL level.

It is now clear how to obtain an improved formula, which contains all the information present in the FO approach, as well as in the HQFF approach. Using Eqs. (4) and (6), we write the improved cross section as

$$\left.\frac{d\sigma}{dx_p}\right|_{\text{imp}} = \sum_{i=0}^{2} a^{(i)}(x_p, E, m) \, \bar{\alpha}_s^i + \sum_{n=3}^{\infty} \beta^{(n)}(x) \, (\bar{\alpha}_s L)^n + \bar{\alpha}_s \sum_{n=2}^{\infty} \gamma^{(n)}(x) \, (\bar{\alpha}_s L)^n \,, \tag{8}$$

where the LL and NLL sums now start from $n = 3$ and $n = 2$ respectively, in order to avoid double counting. Formula (8) includes exactly all terms up to the order α_s^2 with mass effects, and all terms of the form $\left(\bar{\alpha}_s L\right)^n$ and $\bar{\alpha}_s \left(\bar{\alpha}_s L\right)^n$, so that it is also correct at NLL level for $E \gg m$. It can be viewed as an interpolating formula: for moderate energies, it is accurate to the order α_s^2, while for very large energies it is accurate at the NLL level.

3 Non-perturbative effects

When dealing with the HQFF formalism, we have to face the problem that the initial condition for the evolution of the fragmentation functions, which is computed as a power expansion in terms of $\alpha_s(m)$, contains irreducible, non-perturbative uncertainties of order Λ_{QCD}/m [5]. In addition, we need a model to describe the hadronization phenomena, that is the heavy quark turning into a heavy-flavoured hadron. We assume that all these effects, together with all low-energy ones, are described by a non-perturbative fragmentation function D_{NP}^H, so that we can write the full hadronic cross section $d\sigma^H/dx$, including non-perturbative corrections, as

$$\frac{d\sigma^H}{dx} = \frac{d\sigma^P}{dx} \otimes D_{\text{NP}}^H(x) \ , \tag{9}$$

where $d\sigma^P/dx$ is the partonic differential cross section.

We will parametrize the non-perturbative part of the fragmentation function with the one-parameter-dependent Peterson form [1]

$$D_{\text{NP}}^H(x) = P(x,\epsilon) \equiv N \frac{x\,(1-x)^2}{[(1-x)^2 + x\,\epsilon]^2} \ , \qquad \text{with}: \quad \sum_H \int_0^1 dx\, D_{\text{NP}}^H(x) = 1 \ , \tag{10}$$

where the normalization factor N determines the fraction of the hadron of type H in the final state.

4 Fit to experimental data

By fitting the experimental data with the differential cross section of Eq. (9), we can extract the non-perturbative part of the fragmentation functions.

We consider the data sets for D production obtained by ARGUS [6], at $E = 10.6$ GeV, and OPAL [7], at $E = 91.2$ GeV, and the data for B production obtained by ALEPH [8], at $E = 91.2$ GeV.

Besides the LL and NLL fits (similar to that ones made in Ref. [9]), we present new fits with our improved cross section.

We have fitted the data by χ^2 minimization, allowing both the value of ϵ and the normalization to float. We have kept $\Lambda_{\text{QCD}}^{(5)}$ fixed to 200 MeV. The results of the fits are displayed in Table 1. The corresponding curves, together with the data, are shown in Figure 1. The full

Table 1: Results of the fit of the non-perturbative ϵ parameter for the Peterson fragmentation function. The value of $\chi^2/$dof is given in parenthesis. The range of the fit is indicated in Figure 1 with small crosses. [*]We have excluded the first three experimental points for the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ fixed-order fit to the OPAL data.

ϵ ($\chi^2/$dof)	α_s fixed order	LL	α_s^2 fixed order	NLL	NLL improved
ARGUS D	0.058 (0.852)	0.053 (2.033)	0.035 (0.855)	0.018 (1.234)	0.022 (1.210)
OPAL D	$0.078^{(*)}(0.706)$	0.048 (1.008)	$0.040^{(*)}(0.769)$	0.016 (1.122)	0.019 (1.066)
ALEPH B	0.0069 (4.607)	0.0061 (0.137)	0.0033 (2.756)	0.0016 (0.441)	0.0023 (0.635)

improved resummed result of Eq. (8) has been used here. For comparison, we have also plotted the NLL curves computed at the same value of ϵ. The small differences we find in the LL and NLL sectors, with respect to the results of Ref. [9], are due to different range, normalization and adjustment of physical parameters.
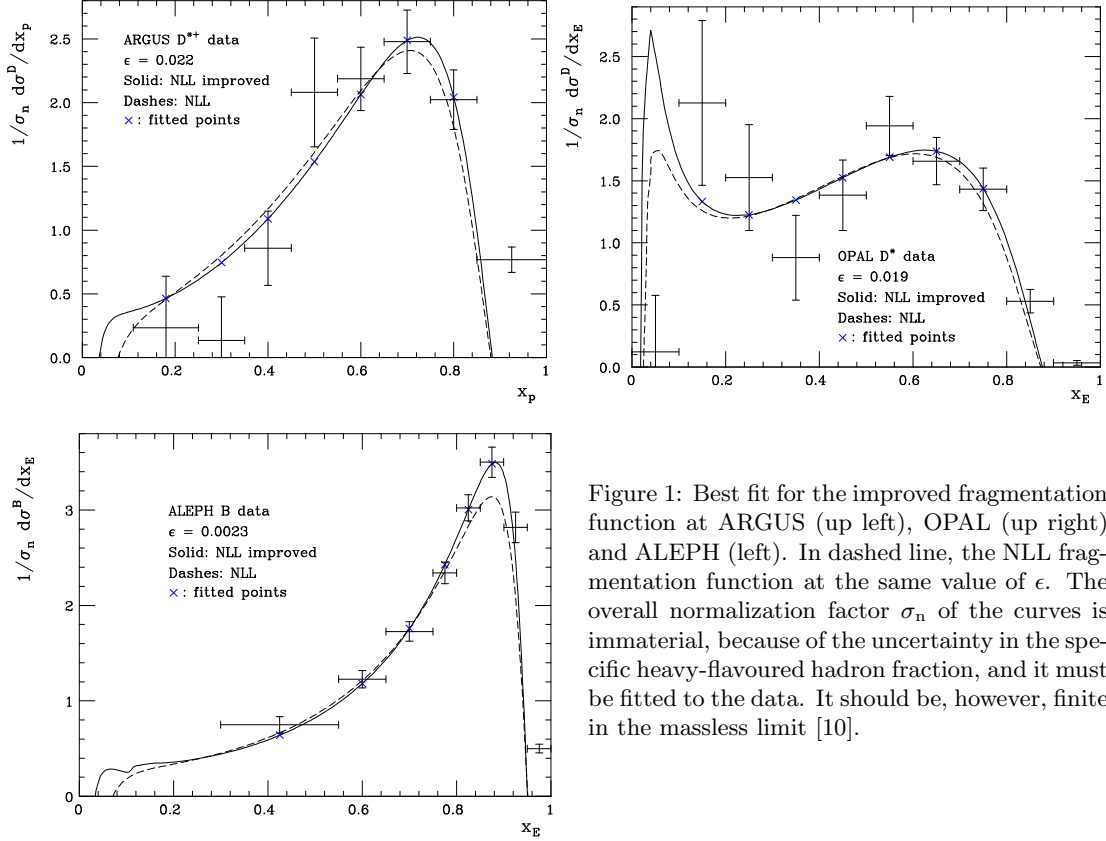
Figure 1: Best fit for the improved fragmentation function at ARGUS (up left), OPAL (up right) and ALEPH (left). In dashed line, the NLL fragmentation function at the same value of ϵ. The overall normalization factor σ_{n} of the curves is immaterial, because of the uncertainty in the specific heavy-flavoured hadron fraction, and it must be fitted to the data. It should be, however, finite in the massless limit [10].

5 Conclusions

In this talk we have presented the results obtained using an improved differential cross section, that contains leading and next-to-leading logarithmic resummation, fixed-order effects up to $\mathcal{O}\left(\alpha_s^2\right)$, and mass effects to the same order, in the fits of present data for heavy-flavour fragmentation functions in e^+e^- collisions. Our finding can be easily summarized as follows. We generally find little differences between our results and the NLL resummed calculation. This indicates that mass effects are of limited importance in fragmentation-function physics in e^+e^- annihilation. On the other hand, our calculation confirms the fact that, when NLL effects are included, the importance of a non-perturbative initial condition is reduced.

Acknowledgments

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